Entropy as The Mother of All

- Our goal was to find the energy $E(T,V)$ as a function of T, V and the pressure, $p(T,V)$.

- Now we found by counting phase space

$$\boxed{S(E,V) = \text{const} + Nk\ln V + \frac{3}{2}Nk\ln E}$$

And identified the derivatives as T and p

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_V = \frac{3Nk}{2E}$$

$$\frac{p}{T} = \left(\frac{\partial S}{\partial V}\right)_E = \frac{Nk}{V}$$

So we see that the entropy determines all:

$$E = \frac{3}{2}NkT$$

$$p = \frac{NkT}{V}$$

If E and p are known experimentally then we can go in reverse, and determine $S(E,V)$ from $E(T,V)$ and $p(T,V)$. We do this next

Entropy of Ideal Gas: Thermodynamic approach

- We have

$$dE = dQ - dW_{out}$$

$$dE = T\,dS - p\,dV$$

Or

$$\boxed{dS = \frac{1}{T}\,dE + \frac{p}{T}\,dV} \qquad (\star)$$

- Now for an ideal gas we found experimentally

$$E = \frac{3}{2}NkT \Rightarrow \frac{1}{T} = \frac{3}{2}\frac{Nk}{E} \qquad (\star\star)$$

$$pV = NkT \Rightarrow \frac{p}{T} = \frac{Nk}{V} \qquad (\star\star\star)$$

So

$$dS = \frac{3}{2}Nk\frac{dE}{E} + Nk\frac{dV}{V}$$

So

$$\boxed{S_{\substack{\text{ideal}\\ \text{gas}}} = \frac{3}{2}Nk\ln E + Nk\ln V + \text{const}} \qquad (\star\star\star\star)$$

MAIG Only

$$= k\ln\left(E^{3N/2}V^{N}\right)$$

So we see that if one measures $p(T,V)$ and $E(T,V)$ we can find S. In a theoretical approach one determines S by counting, and then determines p and E theoretically. We did this first.