Some estimates:

- spacing between atoms/molecules
- typical size of atoms/molecules
- typical speed of atoms/molecules
- typical debroglie wavelength

Later can discuss typical distance between collisions l_{mfp} and typical time between collisions τ_R.

① Typical Spacing:

- We have $PV = n_{ml} RT$ so we can find the volume per particle, V/N, and the typical spacing is $l_0 = (V/N)^{1/3}$. The volume is

$$\boxed{V = \left(\frac{n_{ml} RT}{P}\right) \simeq 22 \text{ Liters}}$$

1 Liter $= 1000 \text{ cm}^3$

* For one mole at, at standard temperature $T_{STP} \equiv 273°K$, at standard pressure, $P_{STP} \equiv 1 \text{bar} \equiv 10^5 \frac{N}{m^2} \simeq 1 \text{atm}$.

So $l_0 \equiv \left(\frac{V}{N_A}\right)^{1/3} \equiv$ typical disance

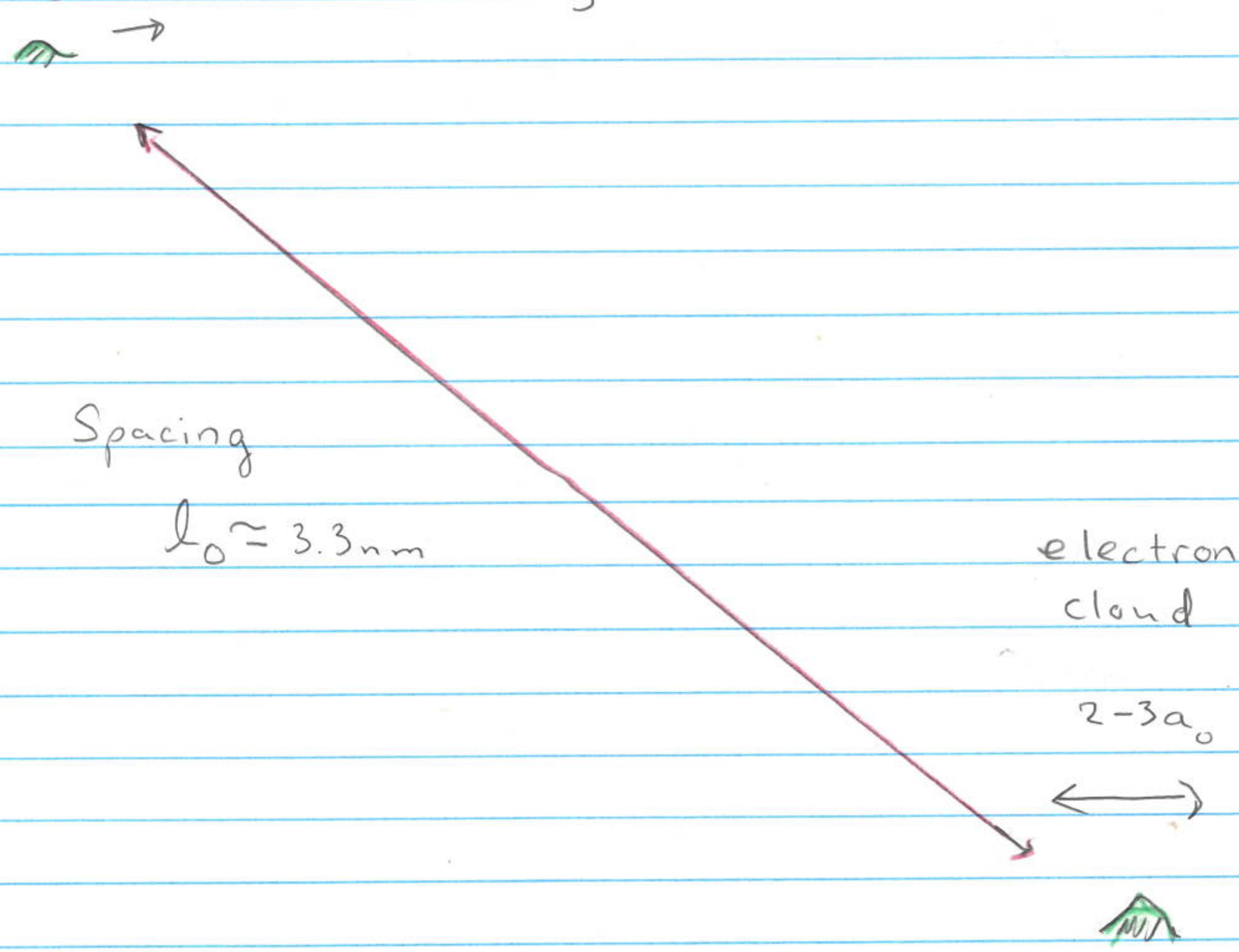
$$\boxed{l_0 \simeq 3.33 \text{ nm}}$$

* STP is "Standard Temperature Pressure", $T = 273°K$, $P = 1 \text{bar} = 10^5 \frac{N}{m^2} \simeq 1 \text{atm}$

(2) The typical size of an atom is the Bohr Radius

$$a_0 = 0.5\ \text{Å}$$ or slightly larger for Larger atoms. For molecules the typical bond length is $1 \sim 2$ Å. $1\ \text{Å} = 0.1\ \text{nm}$

So you should have the picture (which is roughly to scale) in your head:



3 Typical Speeds.

- Take a mono-atomic ideal gas (MAIG).

All of the energy is a result of the translational KE of ideal gas:

means average, the bar also means ave

$$\boxed{\frac{U}{N} = \frac{3}{2} k_B T = \left\langle \frac{1}{2} m \vec{v}^{\,2} \right\rangle} = \frac{1}{2} m \langle v^2 \rangle = \frac{1}{2} m \overline{v^2}$$

$\vec{v}^{\,2} = v_x^2 + v_y^2 + v_z^2$ is the square of the velocity

The brackets denotes an average over the atoms of the gas, i.e molecule 1 has $\vec{v}_1$, molecule 2 has $\vec{v}_2$ etc

$$\langle \vec{v}^{\,2} \rangle \equiv \frac{1}{N} \sum_i \vec{v}_i^{\,2} \equiv \overline{v^2}$$

Thus

$$\boxed{\langle \vec{v}^{\,2} \rangle = \left(\frac{3 k_B T}{m} \right)}$$

$$\langle v_x^2 + v_y^2 + v_z^2 \rangle = \langle v_x^2 \rangle + \underbrace{\langle v_y^2 \rangle + \langle v_z^2 \rangle}_{\text{these are equal}} = \frac{3 k_B T}{m}$$

i.e. $\overline{v_x^2} = \dfrac{k_B T}{m}$

the "root-mean-square" velocity is the square root of this

$$\boxed{V_{rms} \equiv \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3 k_B T}{m}}}$$

Take He which has 2 protons + 2 neutrons. Take STP, so $T = 273°K$. Find V_{rms} without looking up the numbers

$$V_{rms} = \sqrt{\frac{3 N_A k_B T}{m N_A}} = \left(3 \frac{8.32 J}{4 g} (273) \right)^{1/2} = \underline{1300 \, m/s}$$

we used

$$m N_A = 4 g, \quad N_A k_B = R = 8.32 J.$$

Notice that this is of order of the speed of sound in air, $c_s = 330 m/s$. It is somewhat higher than this, reflecting the fact that He is a light atom.

4 Debroglie Wavelength

$\lambda_{th} \sim \frac{h}{p}$ now $p \sim mv \sim m\sqrt{\frac{k_B T}{m}} \sim \sqrt{m k_B T}$

So

$$\boxed{\lambda_{th} \sim \frac{h}{(m k_B T)^{1/2}}}$$

- The book defines (based on calculations)

$\lambda_{th} \equiv \frac{h}{(2\pi m k_B T)^{1/2}}$, but the $\sqrt{2\pi}$ here

is completely arbitrary at this time. To substitute numbers it helps to know a few numbers:

- $m_p c^2 = 938\ \text{MeV} \simeq 1\ \text{GeV}$

mass of proton times c^2

Note also the electron mass $m_e/m_p \simeq 1/2000$ so $m_e c^2 \simeq 0.511\ \text{MeV}$. Protons and neutrons have approx the same mass. Note also the conversion factors

- $\hbar c = 197\ \text{eV nm}$ $hc = 1240\ \text{eV nm}$

So

$$\lambda_{th} = \frac{hc}{(2\pi m_{He} c^2 k_B T)^{1/2}} = \frac{1240\, eV\, nm}{(2\pi \cdot 4 \cdot 1\, GeV\ \frac{1}{44}\, eV)^{1/2}}$$

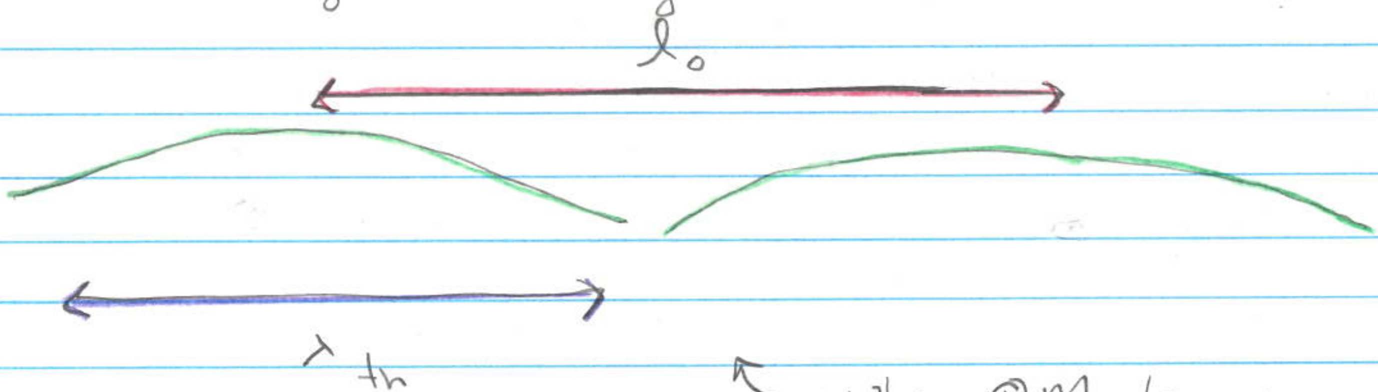
At STP, we have $k_B T = 1/44\ eV$, finally

$$\boxed{\lambda_{th} \simeq 1.0\ \mathring{A}}$$

- So this is really quite short. One might be able to cool to say $10^\circ K$ which

$$\lambda \sim \frac{h}{\sqrt{m k_B T}} \propto \frac{1}{\sqrt{T}} \qquad \frac{\lambda_{10}}{\lambda_{273}} = \sqrt{\frac{273^\circ K}{10^\circ K}}$$

So the wavelength gets longer by $\sqrt{\frac{273}{10}} \simeq 5$ increasing the debroglie wavelength to $\lambda_{th} = 5\ \mathring{A}$. By increasing the density of atoms and cooling further one could hope to reach an interesting regime where the spacings between the atoms is comparable to their Debroglie wavelength



When QM becomes important

In this regime the quantum mechanical charatter of the particles becomes important. We will deal with this at the end of the course, when we discuss Bose Condensates.