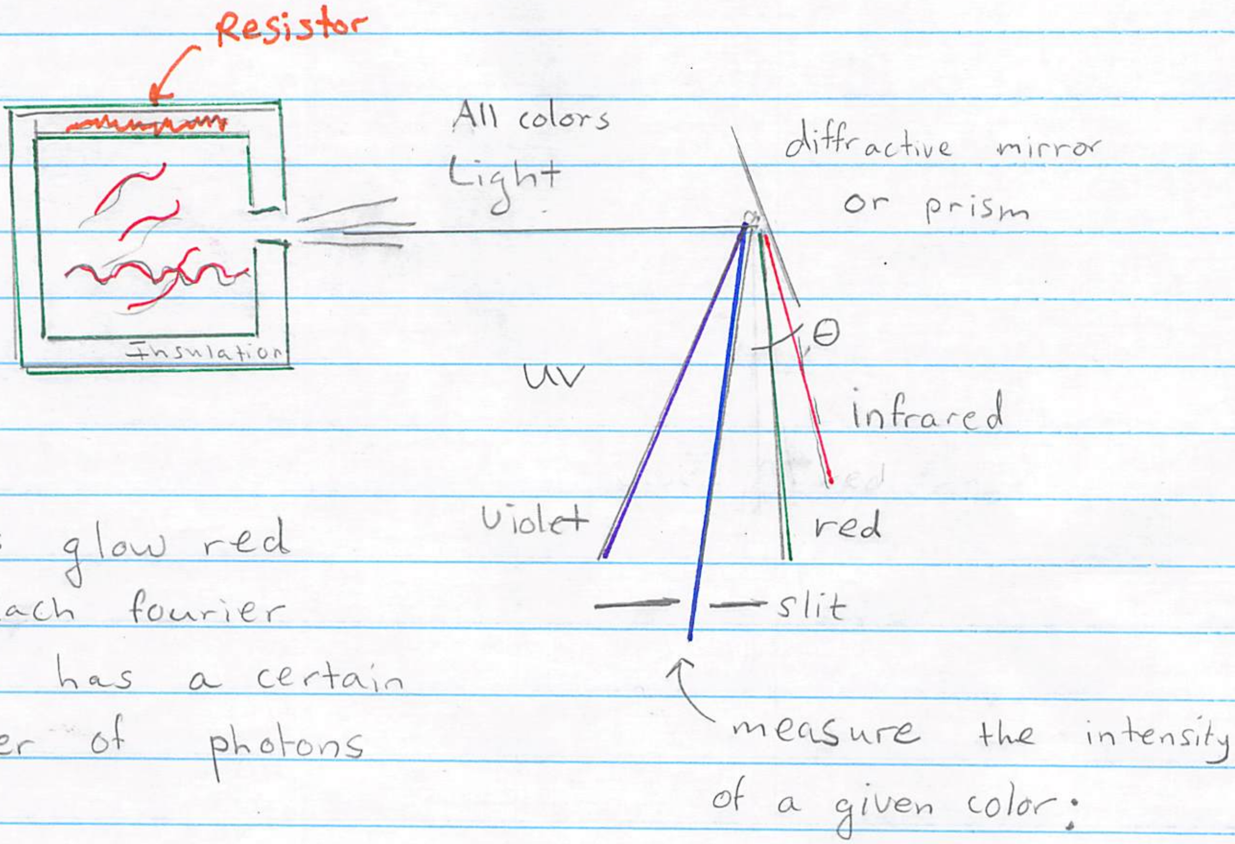
Experiment

On the yield of photons with a specfic frequency



- Objects glow red hot. Each fourier mode has a certain number of photons
- Watch video of oven.
- We just showed that the photon density in the oven is

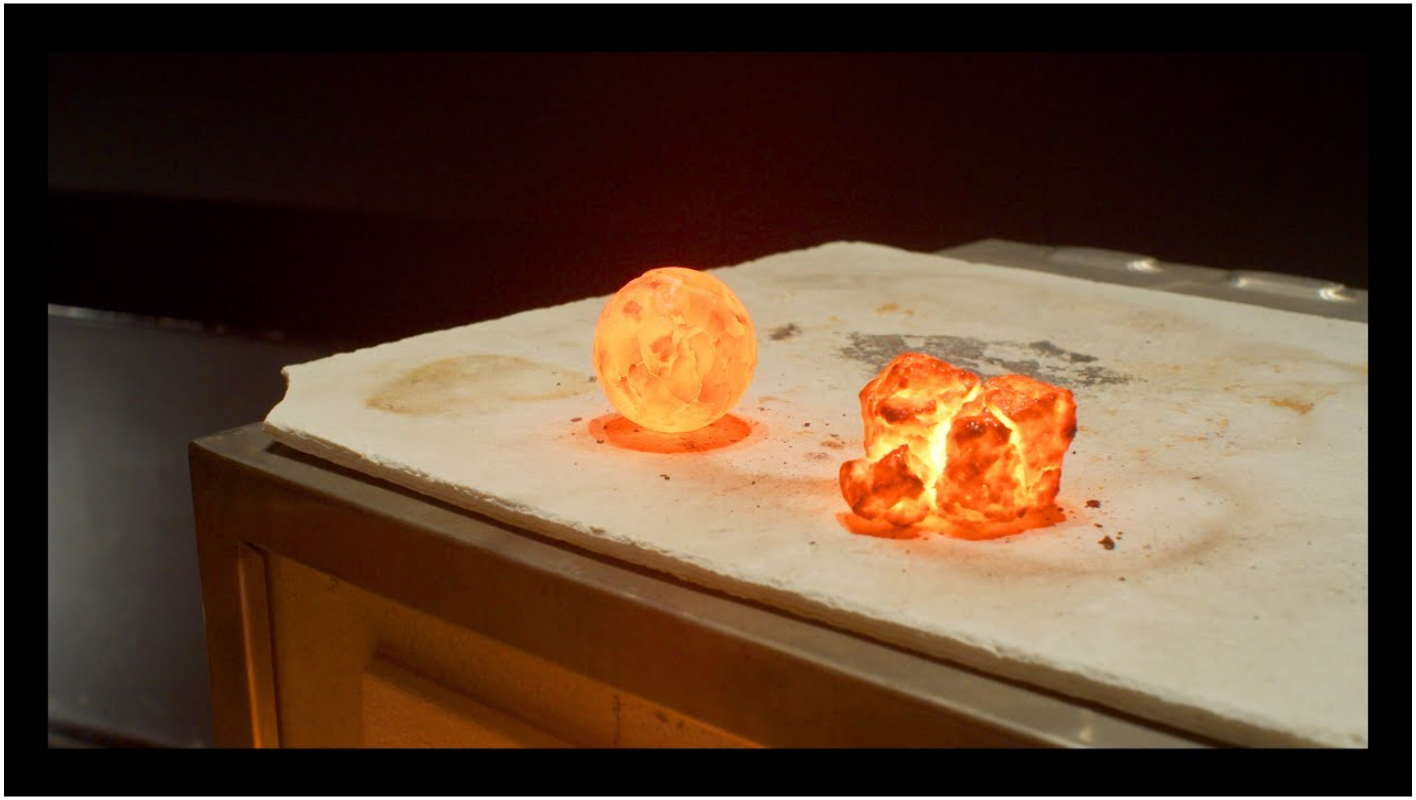
$$n_\gamma = 0.244 \left(\frac{kT}{\hbar c}\right)^3$$

Yields

- We can actually do much more: What is the number of photons per volume with frequency between ω and $\omega + d\omega$, i.e.

$$\frac{dn_\gamma}{d\omega} \qquad \text{note: } \omega = 2\pi f$$

Video: https://www.youtube.com/watch?v=Psvo_XEc784&t=5s



Now recall:

$$N = 2V \int \frac{d^3p}{(2\pi\hbar)^3} \frac{1}{e^{cp/kT}-1}$$

$$d^3p = 4\pi p^2 dp$$

$$N = 2V \int_0^\infty \frac{4\pi p^2 dp}{(2\pi\hbar)^3} \frac{1}{e^{cp/kT}-1}$$

$$n_\gamma = \frac{1}{\pi^2}\frac{1}{\hbar^3}\int_0^\infty \frac{p^2 dp}{e^{cp/kT}-1}$$

So the yield of photons with momentum magnitude between p and p + dp is:

$$dn_\gamma = \frac{1}{\pi^2}\frac{1}{\hbar^3}\frac{p^2 dp}{e^{cp/kT}-1}$$

Now lets convert to frequency

$$\mathcal{E} = cp = \hbar\omega, \quad p = \frac{\hbar\omega}{c}, \quad dp = \frac{\hbar}{c}d\omega$$

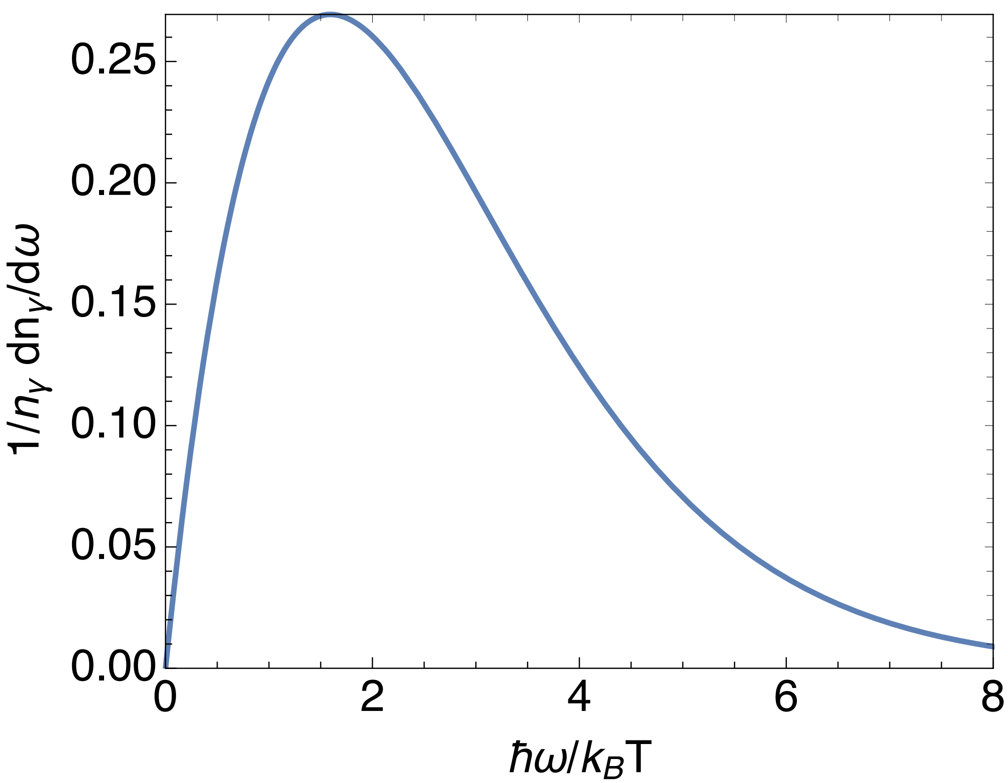
So with this change of variables:

$$\boxed{dn_\gamma = \frac{1}{\pi^2 c^3}\frac{\omega^2}{e^{\hbar\omega/kT}-1}d\omega}$$

Now

$\frac{1}{n_\gamma}\left(\frac{dn_\gamma}{d\omega}\right)$ is shown on the slide

Photon Spectrum



$$\frac{dn_\gamma}{d\omega} \propto \frac{\omega^2}{e^{\hbar\omega/k_B T} - 1}$$

• The curve is maximized when

$$\frac{\hbar\omega}{k_B T} \simeq 1.6 \qquad \text{so the}$$

So the most probable photon energy is $\hbar\omega \simeq 1.6\, k_B T$

Energy Of The Photon Gas

• The mean energy of the photon gas can be determined from the energies of each mode

$$E_p = \bar{n}_{BE}\, \varepsilon(p)$$

Then

$$E = \sum_{\text{moder}} n_{BE}\, \varepsilon(p)$$

$$= 2\int \frac{V\, d^3p}{(2\pi\hbar)^3} \frac{\varepsilon(p)}{e^{\beta\varepsilon(p)} - 1} \qquad \text{use } d^3p = 4\pi p^2\, dp$$

and $\varepsilon(p) = cp$

$$= \frac{V}{\pi^2 \hbar^3} \int_0^\infty \frac{p^2\, dp\, (cp)}{e^{cp/kT} - 1}$$

• Again, measure p in units of $p_0 = kT/c$ & $cp_0 = kT$

$$E = V\left(\frac{p_0}{\hbar}\right)^3 cp_0 \left[\frac{1}{\pi^2}\int_0^\infty \frac{u^3\, du}{e^u - 1}\right]$$

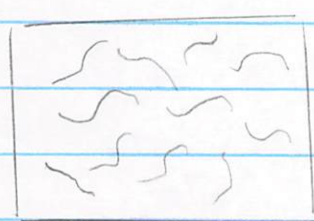
The integral can be done analytically

$$\frac{E}{V} = \left(\frac{p_o}{\hbar}\right)^3 c p_o \left(\frac{\pi^2}{15}\right) \qquad \text{now} \quad c p_o = k_B T$$

$$= \left(\frac{k_B T}{\hbar c}\right)^3 k_B T \left(\frac{\pi^3}{15}\right) \qquad \text{note} \quad \frac{\pi^2}{15} \simeq 0.66$$

- So the energy density is :

$$u = 0.66 \frac{k_B T}{\lambdabar_o^3} \propto T^4 \qquad \text{note} \quad u \sim n_\gamma k_B T$$



- Each photon volume of order $\lambdabar_o^3$ and energy of order $k_B T$

Energy Per Frequency

The energy per volume is

$$u = \frac{c}{\pi^2 \hbar^3} \int_0^\infty \frac{p^3 \, dp}{e^{cp/kT} - 1}$$

We want $\frac{du}{d\omega}$, the energy density per frequency

So

$$du = \frac{c}{\pi^2 \hbar^3} \frac{p^3}{e^{cp/kT} - 1} dp$$

Writing $p = \frac{\hbar \omega}{c}$ $dp = \frac{\hbar}{c} d\omega$ we find

$$du = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{e^{\hbar\omega/kT} - 1} d\omega$$

So

$$\boxed{\frac{du}{d\omega} = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{e^{\hbar\omega/kT} - 1}}$$

energy density per frequency

The Cosmic Microwave Background

- Many years ago, around 370,000 years after the big bang the electrons and protons recombined to make neutral hydrogen. The temperature was around 3000°K. Over the intervening 15 billion years the universe expanded, and the photons from that epoch got red shifted effectively cooling off; every wavelength gets elongated by the same factor.

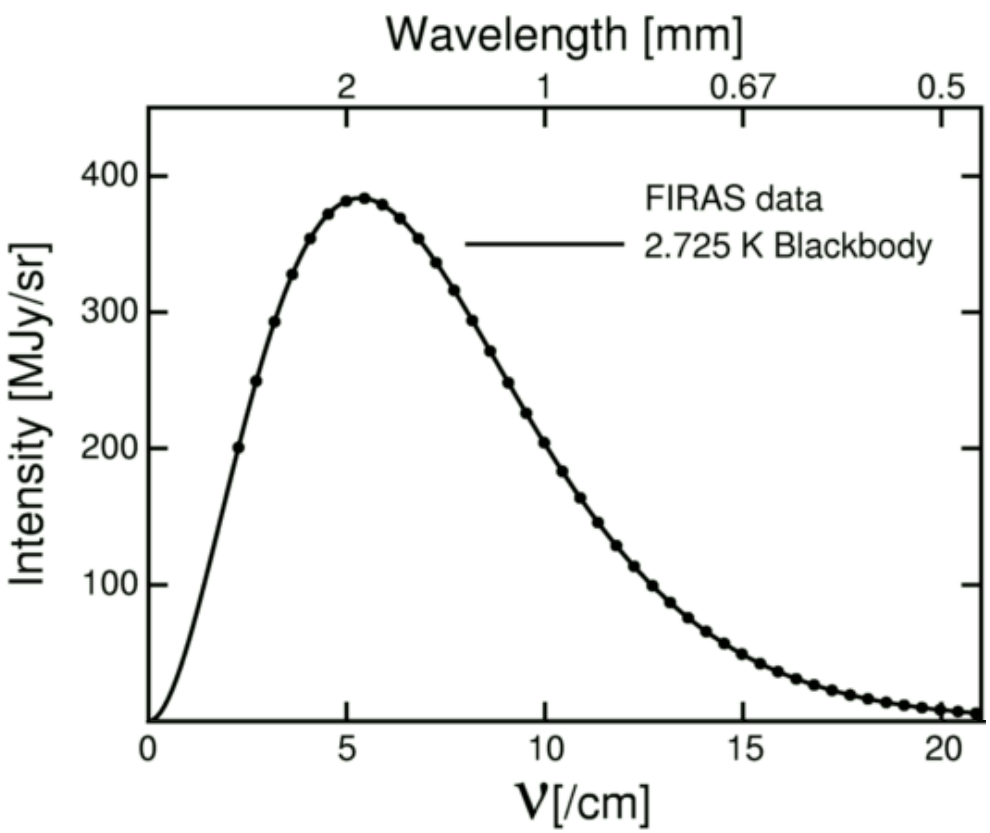
- What is observed is a background spectrum of microwave photons at a temperature of 2.725 °K

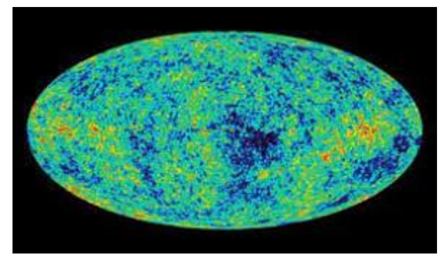
- What is observed in all directions of the sky is the best black body spectrum everv seen.

See Slide:

By fitting the blackbody curve we find $T = 2.725\ K$

The cosmic microwave background





The intensity proportional to :

$$I \propto \frac{\nu^3}{e^{h\nu/k_B T} - 1}$$

The frequency is ν and $h\nu = \hbar\omega$