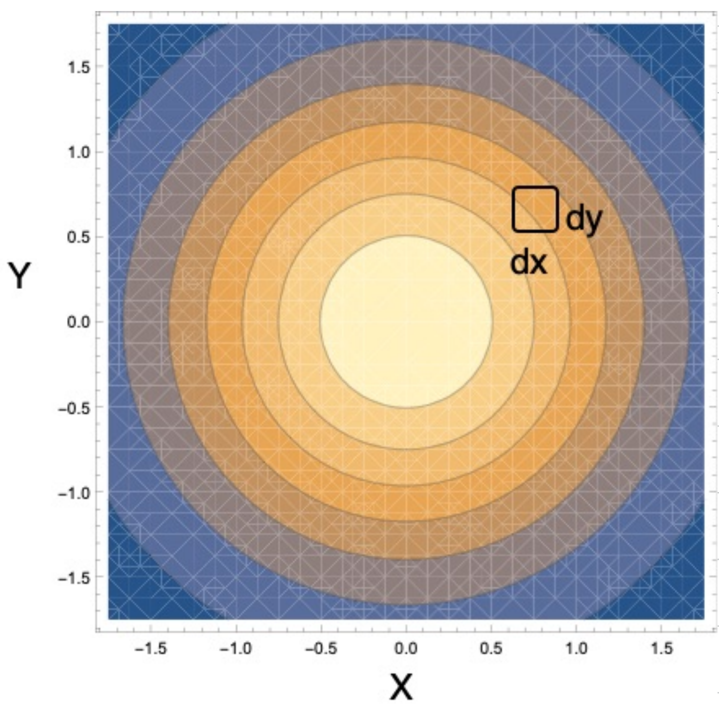
2D Probability Distributions

Now consider a two dimensional probability distibution

$$d\mathcal{P}_{x,y} = P(x,y)\, dx\, dy$$

$d\mathcal{P}_{x,y}$ is the probability of finding the particle in a cell with with x between x and $x+dx$ and y between $y+dy$, or briefly with $[x, dx]$ and $[y, dy]$. This is the probability to be in a bin $dx\,dy$ shown below



Example:

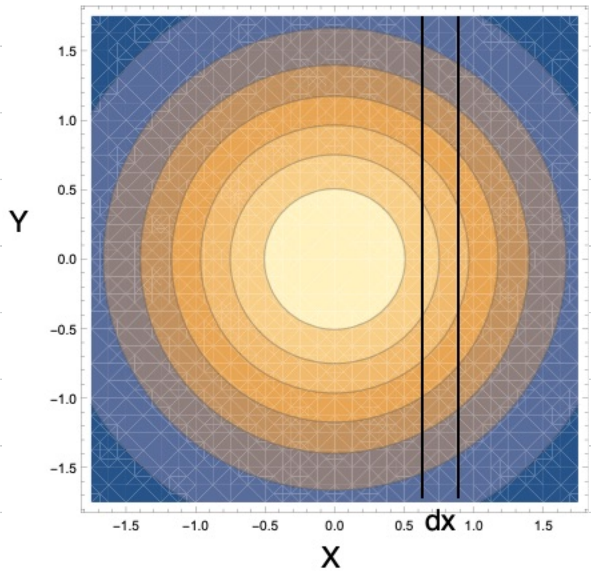
$$\underbrace{d\mathcal{P} = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2}}\, dx\, dy$$

$P(x,y) = \dfrac{d\mathcal{P}}{dx\,dy}$ = units are $\dfrac{1}{m^2}$

- We often only care about one of the variables, say x. Then how to find $d\mathcal{P}_x = P(x)\,dx$ given the probability distribution of x and y?

- Answer:

You "marginalize" over y (aka integrate over y), leaving only a function of x



Any y is ok. Just have to be in bin dx

$$P(x) = \frac{d\mathcal{P}}{dx} = \int \frac{d\mathcal{P}}{dx\,dy}\,dy$$

Example

$$P(x) = \int_{-\infty}^{\infty} \frac{1}{2\pi\sigma^2} e^{-x^2/2\sigma^2} e^{-y^2/2\sigma^2}\,dy$$

$$= \frac{1}{2\pi\sigma^2} e^{-x^2/2\sigma^2} \int_{-\infty}^{\infty} e^{-y^2/2\sigma^2}\,dy$$

$$P(x) = \frac{1}{(2\pi\sigma^2)^{1/2}} e^{-x^2/2\sigma^2}$$