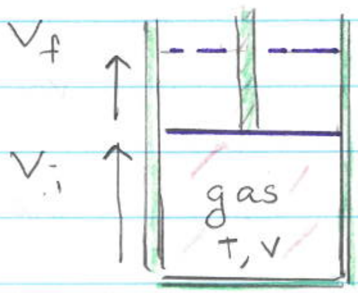
Adiabatic Expansion of Ideal Gas @ Constant C_V

- We will consider a ideal gas with constant specific heat $C_V = \text{const}$, and, $C_p = C_V + Nk_B$, is also constant.

- In an adiabatic expansion we do not allow heat to flow into the cylinder, $\text{đ}Q = 0$.

- Adiabatic expansions are much more common in practice since if the expansion is relatively quick, there isn't time for heat exchange.

Temperature drops as gas expands



← thermally insulated walls, or just do the expansion fast enough (but not too fast) that no heat can be exchanged

$$dQ = 0$$

$$dU = \text{đ}W \quad (1) \qquad \text{Use } \text{đ}W = p\,dV$$

$$C_V\,dT = -p\,dV \quad (2) \quad \text{use ideal gas relation: } dU = C_V\,dT + \overset{0}{\underset{\text{ideal gas}}{\left(\frac{\partial U}{\partial V}\right)_T}} dV$$

$$C_V\,dT = -\frac{Nk_B T}{V}\,dV \quad (3) \quad \text{Use ideal gas EOS.}$$

- So we can integrate this assuming constant specific heat

$$\frac{dT}{T} = -\frac{Nk_B}{C_V}\frac{dV}{V}$$

$$C_p = C_V + Nk_B$$

$$\gamma \equiv \frac{C_p}{C_V} = 1 + \frac{Nk_B}{C_V}$$

$$\frac{dT}{T} = -(\gamma-1)\frac{dV}{V}$$

- Integrating both sides $\ln\frac{T_f}{T_i} = -(\gamma-1)\ln\frac{V_f}{V_i}$ or

$$\boxed{T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}} \quad \text{or} \quad TV^{\gamma-1} \equiv \text{const}$$

So since $pV \propto T$ we find

$$\boxed{p_i V_i^{\gamma} = p_f V_f^{\gamma}} \quad \text{or} \quad pV^{\gamma} = \text{const}$$