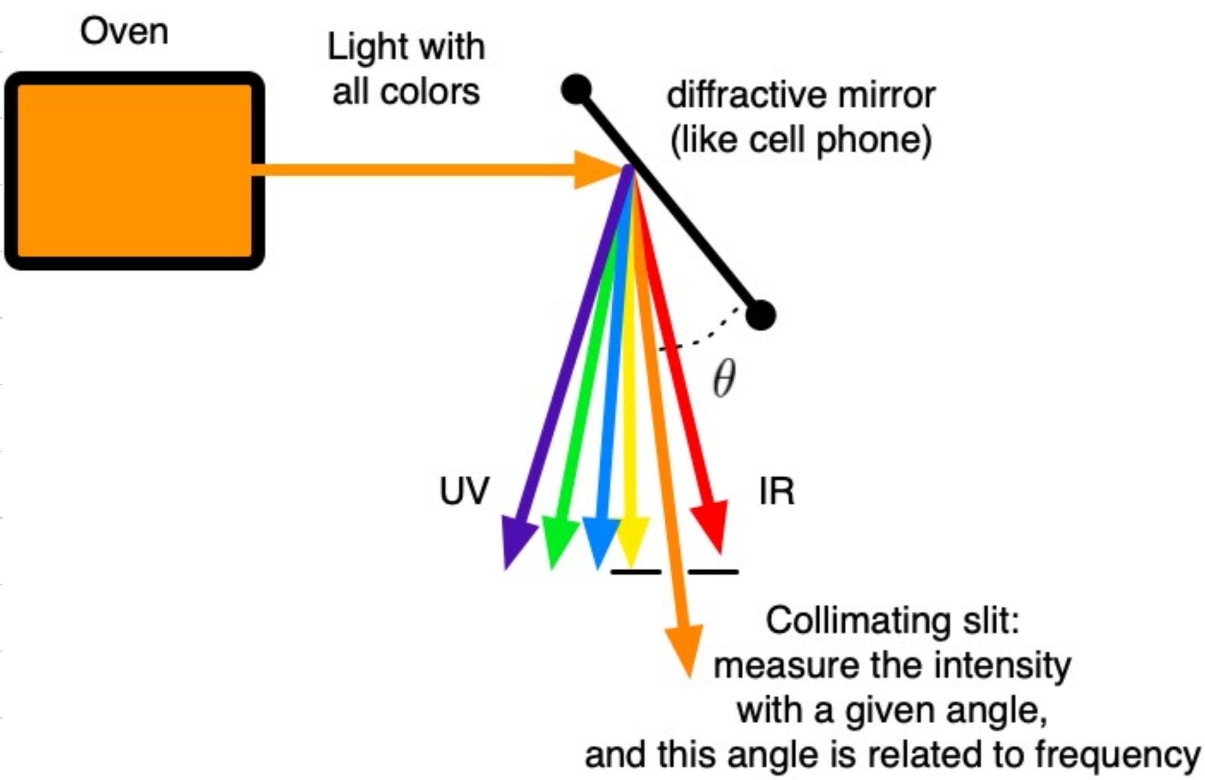
The Blackbody Spectrum

So far we have found the total energy per volume. It is informative to find the energy with frequency between ω and $\omega + d\omega$. This can be measured with a schematic setup shown below:



We want to find:

$$\frac{du_\gamma}{d\omega} = \text{energy per volume per frequency}$$

We already showed that

$$n_\gamma \equiv \frac{N}{V} = \frac{1}{\pi^2\hbar^3}\int_0^\infty \frac{p^2\,dp}{e^{\beta c p}-1}$$ ← Number per Volume

$$u_\gamma \equiv \frac{U}{V} = \frac{c}{\pi^2\hbar^3}\int_0^\infty \frac{p^3\,dp}{e^{\beta c p}-1}$$ ← Energy per Volume

To find the number of photons and energy (per volume) with momentum between p and $p+dp$, just don't do the last integral

$$dn_\gamma \equiv \frac{1}{\pi^2\hbar^3}\,\frac{p^2\,dp}{e^{cp/kT}-1}$$

$$du_\gamma = \frac{c}{\pi^2\hbar^3}\,\frac{p^3\,dp}{e^{cp/kT}-1}$$

Now just change variables

$$cp = \hbar\omega \qquad dp = \hbar\omega/c$$

And so

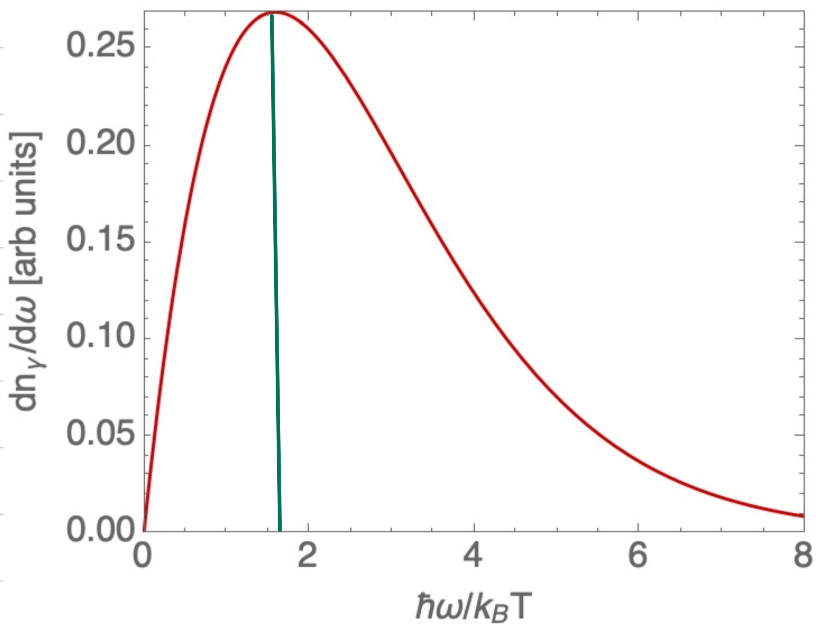
$$dn_\gamma = \frac{1}{\pi^2 c^3}\,\frac{\omega^2\,d\omega}{e^{\hbar\omega/kT}-1}$$

$$du_\gamma = \frac{\hbar}{\pi^2 c^3}\,\frac{\omega^3\,d\omega}{e^{\hbar\omega/kT}-1}$$

↑ energy per volume with frequency between ω and $\omega+d\omega$. Similarly dn_γ is the number of photons per volume in a frequency interval $d\omega$.

The number spectrum is shown below. From the graph we see that the most commonly occuring photon (the maximum of the graph) has frequency

$$\hbar \omega = 1.59\, k_B T$$



The Cosmic Microwave Background

370,000 years after the big-bang, electrons and protons recombined, making neutral Hydrogen. The temperature was around 3000 °K.

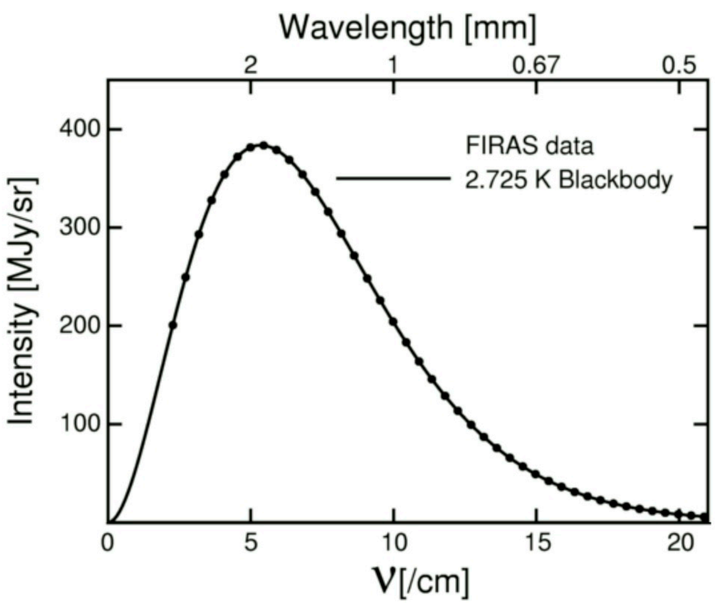
Since light does not scatter off neutral Hydrogen, the light from that epoch has been flying freely for the next 15 Billion years. It gets red-shifted by the expansion, effectively cooling off to 2.725 °K, as each wavelength gets stretched by the same factor.

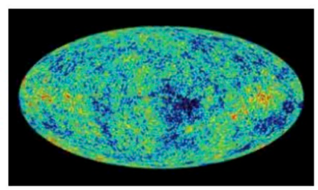
What is observed in all directions of the sky is the best blackbody radiation spectrum ever seen

See Slide Below:

By fitting the blackbody curve we find $T = 2.725\,°K$

The cosmic microwave background





The intensity proportional to :

$$I \propto \frac{\nu^3}{e^{h\nu/k_B T} - 1}$$

The frequency is ν and $h\nu = \hbar\omega$