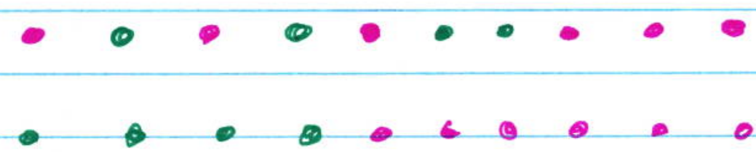
## Combinatorics

- Consider the 10 atoms shown below. Four of them have been "activated"

- There are many ways to activate four some of which are shown below



$N = 10$ sites

$r = 4$ $\qquad$ $N - r = 6$

- The total number ways of selecting 4 sites out of 10 is

$$^{10}C_4 \equiv \frac{10!}{4!\,6!} \qquad \text{"10 choose 4"}$$

or selecting $r$ out of $N$ is

$$^{N}C_r = \frac{N!}{r!\,(N-r)!} \qquad \longleftarrow \text{This is called a binomial coefficient}$$

That's because there are $N!$ rearrangements. But, rearrangements wich shuffle the green dots do not lead to a new selection. There are $r!$ of these. Similarly there $(N-r)!$ rearrangements of the pink dots. So the total number of possible selections is

$$ {}^N C_r = \frac{N!}{r!\,(N-r)!} $$

This generalizes. suppose we have N objects
$N_A$ are A, $N_B$ are B, $N_C$ are C

A A B C B C B C A A B B

Then the number of distinct rearrangements is

$$ \frac{N!}{N_A!\,N_B!\,N_C!} \qquad \text{with} \quad N = N_A + N_B + N_C $$

called a multinomial coefficient