Big Numbers

- Avagadro's number is big

$$N_A = 6.02 \times 10^{23}$$

- But compare (log means natural log)

$$\log N_A \sim 50 \qquad (54.7 \text{ to be more exact})$$

- What about the number of rearrangements of the molecules in this room

$$N_A!$$

This is exponentially large. We will show in a sec, that

$$\boxed{N! \simeq \left(\frac{N}{e}\right)^N = N^N e^{-N} \leftarrow \text{Stirling Approx}}$$

Thus

$$\boxed{\log N! = N \log N - N \leftarrow \text{Stirling Approx}}$$

So

$$\log N_A! = N_A \overbrace{\log N_A}^{54.7} - N_A \simeq 53.7\, N_A = 3 \times 10^{25}$$

So even the $\log N!$ is a very large number.
Lets call this exponentially large

Proof

$$N! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot N$$

$$\log N! = \log(1) + \log(2) + \log(3) + \ldots \log N$$

- This sum of logs can be replaced by an integral if N is large (see figure)

$$\log N! \simeq \int_1^N dx \log x$$

by parts

$$\simeq x \log x - x \Big|_1^N$$

$$\boxed{\begin{aligned} \log N! &\simeq N \log N - N \\ N! &\simeq N^N e^{-N} \end{aligned}}$$

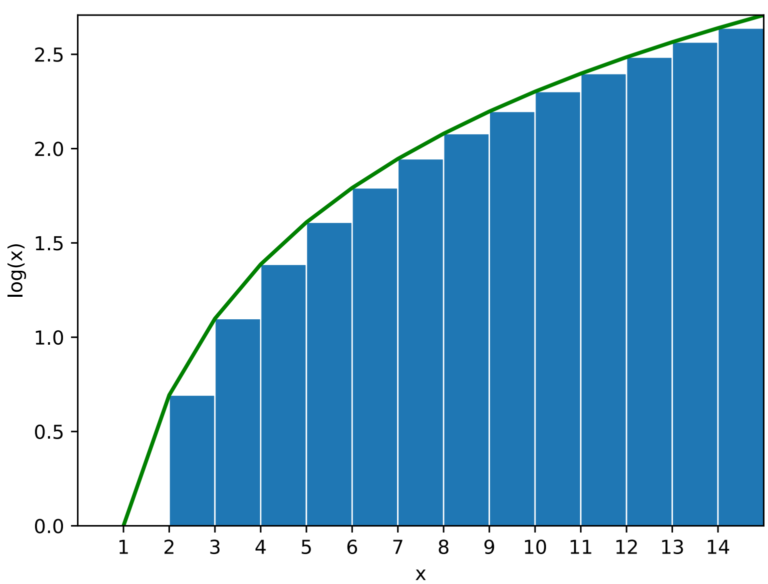
or since $N! = e^{\log N!}$

- You can find a better approximation, if you work harder (see book), which gives

$$N! = N^N e^{-N} \sqrt{2\pi N}$$

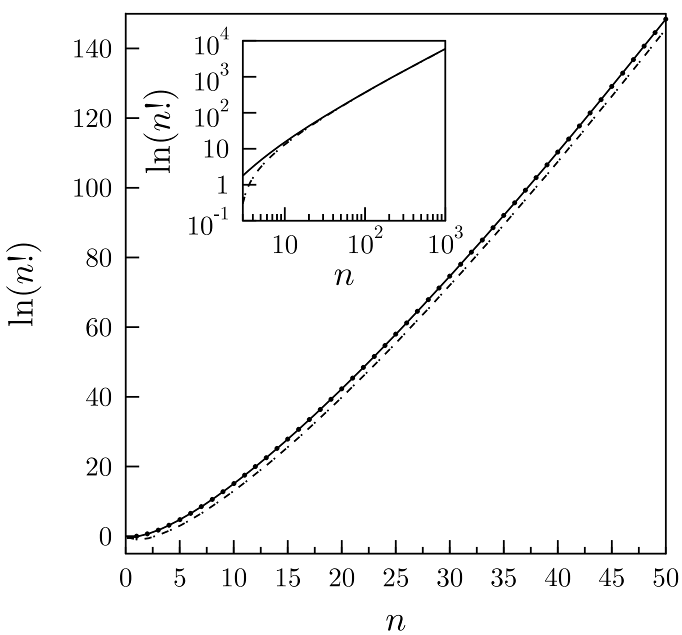
But we will not generally need the $2\pi N$.

Deriving the Stirling approximation:



Replace the sum with integral

Accuracy of Stirling



- Points: $\log(n!)$
- Dashed: $n \log n - n$
- Solid: $\log(n^n e^{-n}\sqrt{2\pi n})$

We will used the dashed