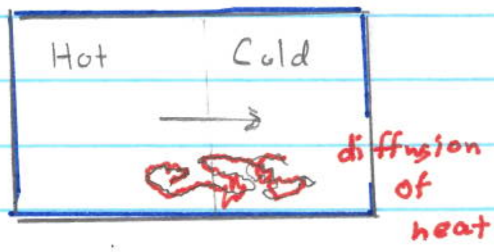
Heat and Specific Heats



- The transfer of energy, aka heat Q, from the hot to the cold

- The transfer is a diffusive process, and is very slow. It takes 1 millisec for sound to propagate across the box, but seconds or more to diffuse across the box.

- An important quantity is the heat capacity

$$C \equiv \frac{đQ}{dT} \quad \text{so} \quad đQ = C\,dT$$

$C\,dT$ is heat required to raise dT.

- The heat capacity grows with the system size (the ocean has a large heat capacity). We usually quote the specific heat per mol or per kilo, i.e. the specific heat for 1mol of substance. The book uses $c = C/\text{Mass}$ for the specific heat per kilo

- For one mole, $C^{1\text{mol}}$ is of order $R = 8.32 \frac{J}{\text{mol}\ {}^\circ K}$

- There are two kinds of specific heats. First consider holding the volume fixed and heating the system. (See slide a)

$$C_V \equiv \frac{đQ_V}{dT}$$ ← notates that V is const

- Then since V is constant the work done is zero. So

$$dU_V = đQ_V + đW \quad (dV = 0)$$

$$\boxed{C_V = \frac{dU_V}{dT} = \left(\frac{\partial U}{\partial T}\right)_V}$$

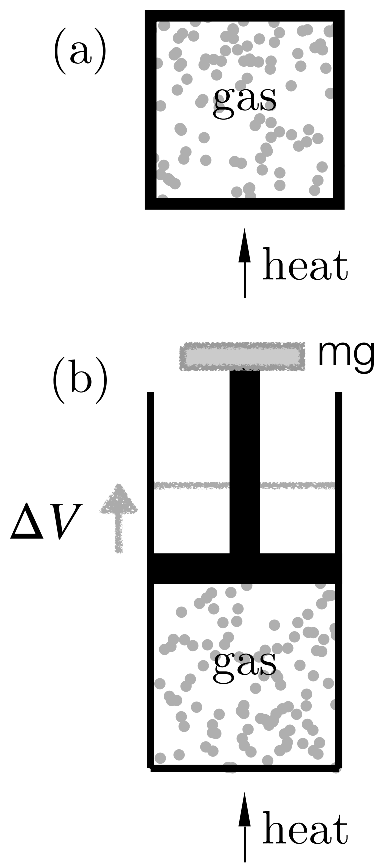
derivative of U with V constant is a partial derivative

- Now consider heating the gas at constant pressure (see slide b). This can be accomplished by allowing the gas to expand as it is heated.

$$\boxed{C_p \equiv \frac{đQ_p}{dT}}$$

It is clear that C_p is greater than C_V, since for the same amount of heat the gas does work. And thus not all of the heat leads to a temperature rise. We now show that $\boxed{C_p = C_V + Nk_B}$ for an IG:

Specific heats at constant volume and pressure



Constant volume: add heat,
and temperature goes up

$$C_v = \left(\frac{đQ}{dT}\right)_V$$

Constant pressure: add heat,
the gas expands doing work, and
temperature goes up, but not as much

$$C_p = \left(\frac{đQ}{dT}\right)_p$$

Proof

- From the first law

$$dQ = dU - dW = dU - (-P\,dV) = dU + P\,dV$$

- Now for an ideal gas U is not a fcn of Volume

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \cancel{\left(\frac{\partial U}{\partial V}\right)_T}^{\,0} dV = C_V\, dT$$

← ideal gas only

- Also for an ideal gas at constant pressure

$$V = \frac{NkT}{P} \Rightarrow dV = \frac{Nk}{P}\,dT$$

← const pressure ideal gas

So for

$$dQ_P = C_V\, dT + Nk\, dT$$

Dividing by dT

$$\frac{dQ_P}{dT} \equiv \boxed{C_P = C_V + Nk_B}$$

Ideal Gas Only

For one mol of substance $N = N_A$ and $N_A k_B = R$ so:

$$\boxed{C_P^{1mol} = C_V^{1mol} + R}$$

Examples Gasses

- So for a mono-atomic ideal gas (MaIG) and a a diatomic ideal gas (DAIG) we have

$$U = \begin{cases} \frac{3}{2} NkT & (MAIG) \\ \frac{5}{2} NkT & (DAIG) \end{cases}$$

- So for one mole $N = N_A$ and $N_A k_B = R$! we have

$$C_V^{1mol} = \begin{cases} 3/2\, R & (MAIG) \\ 5/2\, R & (DAIG) \end{cases}$$

And

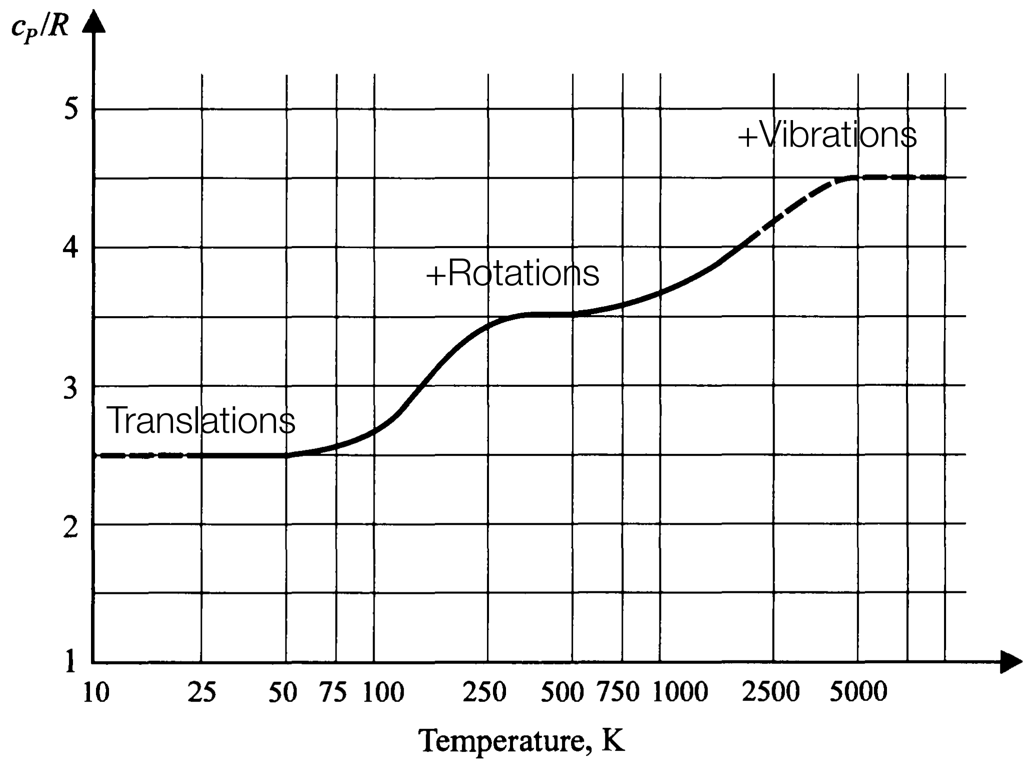
$$C_p^{1mol} = \begin{cases} 5/2\, R & (MAIG) \\ \frac{7}{2} R & (DAIG) \end{cases}$$

- Looking at the measured specific heats for Hydrogen H_2 (see below) we see that at low temperatures $T \sim 50°K$ hydrogen is like a mono-atomic gas $C_p = 5/2\, R$, but for room temperatures $T \sim 290°K$ it behaves as a diatomic gas $C_p = \frac{7}{2} R$.

- **Adiabatic Index**

The adiabatic index is $\gamma \equiv \frac{C_p}{C_V}$

Specific Heat of H_2



For an ideal gas $C_p = C_v + Nk$

$$\gamma = 1 + \frac{Nk}{C_v}$$

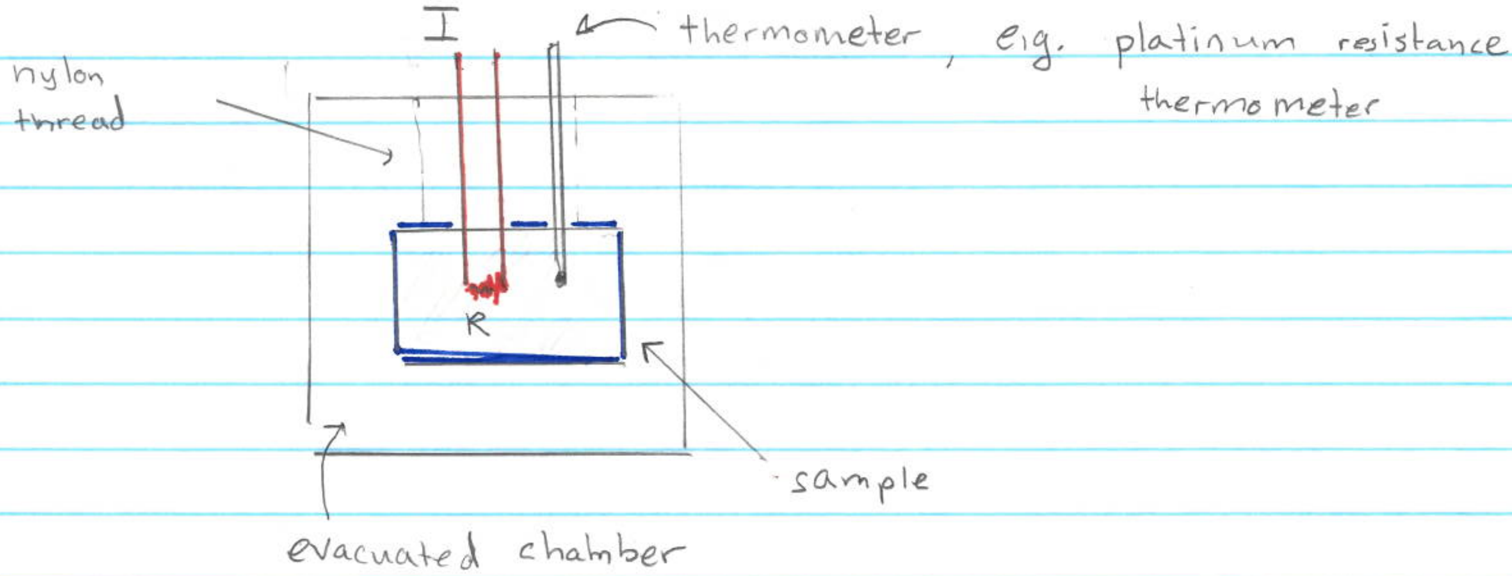
So

$$\gamma = \begin{cases} 5/3 = 1.66 & \text{(MAIG)} \\ 7/5 = 1.4 & \text{(DAIG)} \end{cases}$$

For most real gasses γ is nearly constant and close to these values. For solids C_p and C_v are nearly equal (since solids don't expand much upon heating) and $\gamma \simeq 1$.

Solids

- We can measure specific heats of solids as follows. Take a solid sample. Embed a resistor R in the sample:



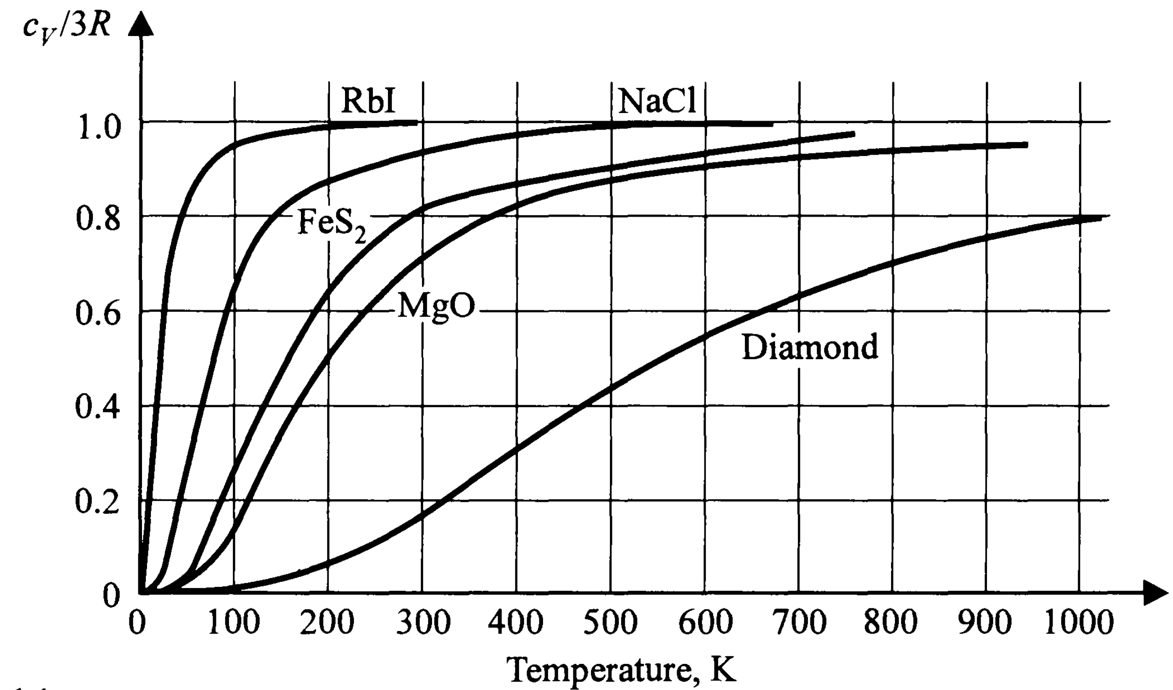
- Turn on a current for a time Δt. The power dissipated is $P = I^2 R$ so the heat delivered is

$$\Delta Q = I^2 R \Delta t$$

And then measure ΔT. This is $C_p = \Delta Q / \Delta T$ since the sample will expand.

- The figure below shows the specific heat per mole for various substances. We see that C_p approaches $3R$ at high temperatures but is smaller at low temperatures. The $3R$ has a simple explanation explored in Homework.

Specific Heats of Solids: (Taken from Zemansky and Dittman)



It is not an exaggeration to say that the goal of the course is to explain these curves!