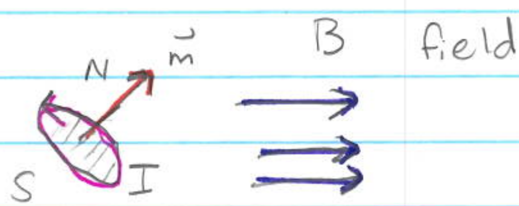
Para Magnets

- We have primarily focussed on simple hydrodynamic systems
- Magnetic Systems provide another good example for our thermodynamics
- First consider a small coil of wire with current I



- each coil of wire acts like a compass needle, with magnetic moment

$$\vec{m} = I \vec{A}$$

magnetic moment — area of coil

The magnetic moment (compass needle) wants to align itsself with $\vec{B}$

$$U = -\vec{m} \cdot \vec{B}$$

potential energy of magnetic moment in magnetic field

- Cosider a system of magnetic moments arranged on a lattice. If applying a magnetic field causes the moments to line up, the thermo-dynamic system is called paramagnetic.

- The magnetization M is the magnetic moment per volume;

$$M = \frac{m}{V}$$ and is intensive, while m is extensive

- The magnetic field H, is related to M via $\vec{B} = \mu_0(\vec{H} + \vec{M})$. The analog of $dU = TdS - pdV$ is

$$dU = TdS - \vec{m} \cdot d\vec{B}$$

- Then magnetic moment is a function of temperature and H, $m(T,H)$. Typically M is quite small and $B \simeq \mu_0 H$. The isothermal magnetic susceptibility is

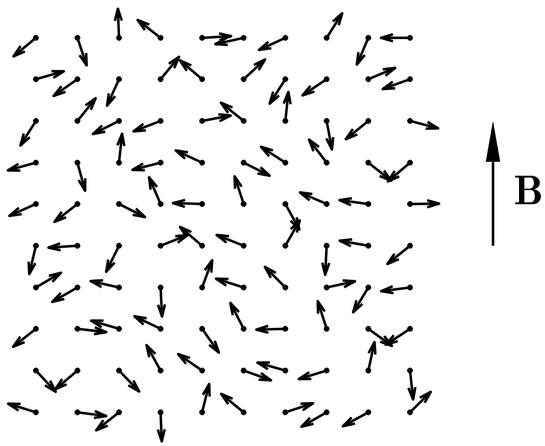
$$\boxed{\chi_T \equiv \left(\frac{\partial \langle M \rangle}{\partial H}\right)_T \simeq \left(\frac{\partial \langle M \rangle}{\partial B}\right)_T}$$

Small H Limit

For small H, $\langle M \rangle \propto H$ and $\boxed{\chi_T^0 \simeq \lim_{H \to 0} \langle M \rangle / H.}$

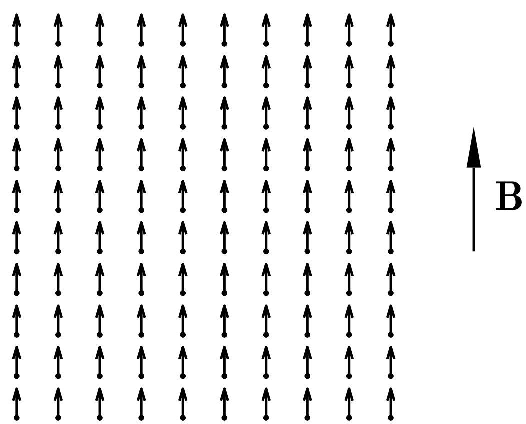
- At high temperature the magnetic moments are all random (the effect of the alligning field is small). See slide

Thus we expect the magnetization to decrease



(a) high temperature

Picture of paramagnetic material



(b) low temperature

as the temperature increases

$$\chi \propto \frac{1}{T}$$

This is known as the Currie Law

Example:

- How much heat is emitted, if the temperature is held fixed, by the magnetic fied is increased by dB

Define:

Note: $dU = T\,dS - \vec{m}\cdot d\vec{B}$

$$F = U - TS$$

$$dF = -S\,dT - m\,dB \longrightarrow \left(\frac{\partial S}{\partial B}\right)_T = \left(\frac{\partial m}{\partial T}\right)_B$$

So

$$dQ = T\,dS = T\left(\frac{dS}{dB}\right)_T dB = T\left(\frac{\partial m}{\partial T}\right)_B dB$$

$$dQ \approx TV\frac{B}{\mu_0}\frac{\partial \chi}{\partial T}\,dB < 0$$

$m \simeq \chi(T)\,HV$

$\frac{\partial \chi}{\partial T} < 0$ since $\chi \propto \frac{1}{T}$

- In general many systems besides magnets and gasses have macroscopic thermodynamic properties. The first step is to identify the variables and the corresponding first law:

$$\underbrace{dW = X}_{\text{generalized force}}\, \underbrace{dx}_{\text{generalize coordinate}}$$

Ex:

dW	X	x
$-p\,dV$	P	V
$-\vec{m}\cdot d\vec{B}$	$\vec{m}$	$\vec{B}$

System with press. and volume, liquids etc.

magnetic systems

A model of a paramagnet

- Consider a solid consisting of N atoms of spin 1/2 a regular rectangular array. The magnetic moment of each atom is $\vec{\mu}_B$, this is proportional to the spin

$$\vec{\mu} = g \frac{e}{2m_p} \vec{S} \quad \leftarrow \pm\hbar/2$$

m_p ← proton mass

g factor dimensionless number -- fudge factor

The reason for this notation is that for a pronon or for an electron going around in a circle, $\vec{\mu} \equiv I\vec{A}$ is $\vec{\mu} = \frac{e}{2m}\vec{L}$

- The spin is either up $\langle S_z \rangle = \hbar/2$ or down $\langle S_z \rangle = -\hbar/2$. Then each atom in a magnetic field in the z-direction is $\vec{\mathcal{E}} = -\vec{\mu} \cdot \vec{B}$

$$\boxed{\mathcal{E}_s = -\mu_B B s} \quad \text{with } s = \pm 1 \text{ and } \mu_B = g\frac{e\hbar}{4m_p}$$

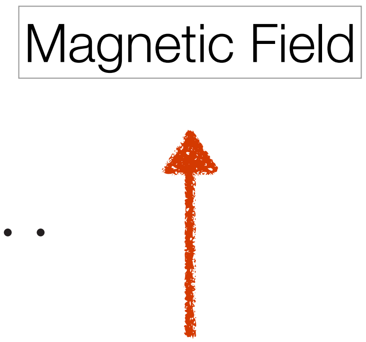
+1 for spin up
-1 for spin down

- Then

$$E = \mathcal{E}_{s_1} + \mathcal{E}_{s_2} + \ldots \mathcal{E}_{s_N}$$

$$Z = \sum_{s_1 s_2 s_3 \ldots s_N} e^{-E/k_B T} = Z_1^N$$

Energetics and States



This state has the lowest energy

$$\cdots \uparrow \cdots$$

But the there are many more states that look like this

$$\cdots \uparrow\uparrow\downarrow\uparrow\downarrow\downarrow\uparrow\downarrow\downarrow\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow\downarrow\downarrow\downarrow\uparrow \cdots$$

At a fixed temperature and magnetic field we are to minimize, $F = U - TS$

- Where

$$Z_1 = \sum_{s=-1,1} e^{-\varepsilon_s / k_B T}$$

$$= e^{-\mu_B B \beta} + e^{+\beta \mu_B B} = 2\cosh(\beta \mu_B B)$$

Aside

$$\cosh x \equiv \frac{e^{x} + e^{-x}}{2} = \cos(ix) \qquad \frac{d \cosh x}{dx} = \sinh x$$

$$\sinh x \equiv \frac{e^{x} - e^{-x}}{2} = \frac{\sin(ix)}{i} \qquad \frac{d}{dx}\sinh x = \cosh x$$

note all signs are +

- Then

$$F_{TOT} = -k_B T \ln Z_{TOT} = -N k_B T \ln Z_1$$

$$F_{OT} = -N k_B T \ln\left[2\cosh(\beta \mu_B B)\right]$$

- Then

$$dE = T\,dS - m\,dB$$

$$dF = d(E - TS) = -S\,dT - m\,dB$$

So

$$m = -\left(\frac{\partial F}{\partial B}\right)_T = N \mu_B \tanh(\beta \mu_B B)$$

- Given m we can compute the magnetization

$$\mathcal{M} = \frac{m}{V} = n\mu_B \tanh(\beta\mu_B B)$$ (see slide)

- At small magnetic field, we use $\tanh x \simeq x + O(x^3)$ and thus

$$\mathcal{M} \simeq n\mu_B \beta \mu_B B = \frac{n\mu_B^2}{k_B T} B$$

The susceptiblitity is

$$\chi_T^0 \simeq \lim_{H \to 0} \frac{M}{H}$$

- Now the magnetization is small and $B = \mu_0(H + M)$ is approx, $B \simeq \mu_0 H$ or $H \simeq \frac{B}{\mu_0}$ and thus

$$\boxed{\chi \simeq \frac{n\mu_0\mu_B^2}{k_B T} \propto \frac{1}{T}}$$

Thus we have proved the Curie's Law $\chi \propto 1/T$ for this system