

# 1 Integrals

**Bose and Fermi:**

$$\int_0^{\infty} dx \frac{x}{e^x - 1} = \frac{\pi^2}{6} \quad (1)$$

$$\int_0^{\infty} dx \frac{x^2}{e^x - 1} = 2\zeta(3) \simeq 2.404 \quad (2)$$

$$\int_0^{\infty} dx \frac{x^3}{e^x - 1} = \frac{\pi^4}{15} \quad (3)$$

$$\int_0^{\infty} dx \frac{x^4}{e^x - 1} = 24\zeta(5) \simeq 24.88 \quad (4)$$

$$\int_0^{\infty} dx \frac{x^5}{e^x - 1} = \frac{8\pi^6}{63} \quad (5)$$

$$\int_0^{\infty} dx \frac{x}{e^x + 1} = \frac{\pi^2}{12} \quad (6)$$

$$\int_0^{\infty} dx \frac{x^2}{e^x + 1} = \frac{3}{2} \zeta(3) \simeq 1.80309 \quad (7)$$

$$\int_0^{\infty} dx \frac{x^3}{e^x + 1} = \frac{7\pi^4}{120} \quad (8)$$

$$\int_0^{\infty} dx \frac{x^4}{e^x + 1} = \frac{45}{2} \zeta(5) \simeq 23.33 \quad (9)$$

$$\int_0^{\infty} dx \frac{x^5}{e^x + 1} = \frac{31\pi^6}{252} \quad (10)$$

**Gamma Function:**

$$\Gamma(z) \equiv \int_0^{\infty} x^{z-1} e^{-x} dx \quad (11)$$

with specific results

$$\Gamma(z + 1) = z\Gamma(z) \quad \Gamma(n) = (n - 1)! \quad \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} \quad (12)$$

**Gaussian Integrals:**

$$I_n = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx e^{-x^2/2} x^n \quad (13)$$

with specific results

$$I_0 = 1 \quad I_2 = 0 \quad I_4 = 3 \quad I_6 = 15 \quad (14)$$